

Subthreshold kaon production and the nuclear equation of state

G. Q. Li and C. M. Ko

Cyclotron Institute and Physics Department

Texas A&M University, College Station, Texas 77843

Abstract

We reexamine in the relativistic transport model the dependence of kaon yield on the nuclear equation of state in heavy ion collisions at energies that are below the threshold for kaon production from the nucleon-nucleon interaction in free space. For Au+Au collisions at 1 GeV/nucleon, we find that the kaon yield measured by the Kaos collaboration at GSI can be accounted for if a soft nuclear equation of state is used. We also confirm the results obtained in the non-relativistic transport model that the dependence of kaon yield on the nuclear equation of state is more appreciable in heavy ion collisions at lower incident energies. We further clarify the difference between the predictions from the relativistic transport model and the non-relativistic transport model with a momentum-dependent potential.

About ten years ago, Aichelin and Ko [1] pointed out that in heavy ion collisions at incident energy per nucleon that was below the kaon production threshold in nucleon-nucleon interaction in free space (which is about 1.58 GeV) kaon production was sensitive to the nuclear equation of state (EOS) at high densities. In the non-relativistic Boltzmann-Uehling-Uhlenbeck (BUU) model, they found that in central heavy ion collisions at an incident energy of 0.7 GeV/nucleon the kaon yield obtained with a soft EOS (compressibility $K=200$ MeV) was about 2-3 times larger than that obtained with a stiff EOS ($K=380$ MeV). This finding was later confirmed in calculations based on the quantum molecular dynamics (QMD) [2,3].

The determination of the nuclear EOS at high densities has been one of the main motivations for recent experimental measurements of kaons in heavy ion collisions around 1 GeV/nucleon by the Kaos collaboration at GSI [4,5]. Since the experimental data have become available, there has been a resurgence of theoretical studies on kaon production in heavy ion collisions, based on both non-relativistic [6–8] and relativistic [9,10] transport models. Huang *et al.*, [6] have carried out the first comparison of theoretical results, obtained with the QMD model, with the experimental data from Au+Au collisions at 1 GeV/nucleon. Good agreements with the experimental data have been obtained when a soft EOS is used in the model. With a stiff EOS, their results are about a factor of two below the experimental data. These findings have recently been confirmed by Hartnack *et al.*, [7], using also the QMD model. A similar calculation has been carried out by Li [8] in the non-relativistic BUU approach. Again, a soft EOS has been found to give reasonable agreements with the experimental data.

In both BUU and QMD calculations, the nuclear EOS is modeled by a simple Skyrme parameterization. In this model, the energy density of the nuclear matter at density ρ is given by

$$\mathcal{E} = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma + 1} \frac{\rho^{\gamma+1}}{\rho_0^\gamma} + \frac{4}{(2\pi)^3} \int_0^{k_F} d^3\mathbf{k} (m^2 + \mathbf{k}^2)^{1/2}, \quad (1)$$

where k_F is the Fermi momentum and m is the nucleon mass. The parameters α , β and γ are determined by requiring that in normal nuclear matter one has a saturation density

of $\rho_0 = 0.17 \text{ fm}^{-3}$, a binding energy of 15.6 MeV, and a compressibility of $K=200 \text{ MeV}$ (soft EOS with $\alpha = -356 \text{ MeV}$, $\beta = 303 \text{ MeV}$, and $\gamma = 7/6$) or 380 MeV (stiff EOS with $\alpha = -124 \text{ MeV}$, $\beta = 70.5 \text{ MeV}$, and $\gamma = 2$). These two sets of EOS (i.e., the energy per nucleon as defined by $E/A = \mathcal{E}/\rho - m$) are shown in Fig. 1 by the dashed curves. At $3\rho_0$, the two differ by about 55 MeV. We note that in the Skyrme parameterization, the compressional energy at high densities depends mainly on the second term in Eq. (1), or more specifically, on the magnitude of γ , which is directly proportional to the nuclear compressibility K at the saturation density.

On the other hand, the calculations of Fang *et al.* [9] and Maruyama *et al.* [10] have been carried out in the relativistic transport model where the nuclear EOS is modeled using the non-linear σ - ω model [11]. The energy density of the nuclear matter in this model is given by

$$\begin{aligned} \mathcal{E} = & \frac{g_\omega^2}{2m_\omega^2}\rho^2 + \frac{m_\sigma^2}{2g_\sigma^2}(m - m^*)^2 + \frac{b}{3g_\sigma^3}(m - m^*)^3 + \frac{c}{4g_\sigma^4}(m - m^*)^4 \\ & + \frac{4}{(2\pi)^3} \int_0^{k_F} d^3\mathbf{k} (m^{*2} + \mathbf{k}^2)^{1/2}, \end{aligned} \quad (2)$$

where m^* is the nucleon effective mass. The parameters g_σ , g_ω , b , and c are determined by not only the normal nuclear matter properties, such as the saturation density, the binding energy, and the compressibility, but also the nucleon effective mass. In Ref. [9], we have used two sets of parameters which correspond to the same nucleon effective mass, $m^* = 0.83 m$, but different values of nuclear compressibility, i.e., $K=380 \text{ MeV}$ (with $C_\sigma = (g_\sigma/m_\sigma)m = 11.27$, $C_\omega = (g_\omega/m_\omega)m = 8.498$, $B = b/(g_\sigma^3 m) = -2.83 \times 10^{-2}$, and $C = c/g_\sigma^4 = 0.186$) and $K=200 \text{ MeV}$ (with $C_\sigma = 13.95$, $C_\omega = 8.498$, $B = 1.99 \times 10^{-2}$, and $C = -2.96 \times 10^{-3}$). It has been found in Ref. [9] that the kaon yield at 1 GeV/nucleon obtained with $K=200 \text{ MeV}$ is only about 15% larger than that with $K=380 \text{ MeV}$. This result is inconsistent with the findings from the non-relativistic transport models [6–8], where the difference between the results obtained with the two Skyrme EOS's is about a factor of two.

It has recently been realized that the so-call “stiff” EOS used in Ref. [9] is not as stiff as that in the Skyrme parameterization. As can be seen from Eq. (2), in the relativistic

approach the EOS at high densities largely depends on the vector repulsion from the omega meson. From the Hugenholtz-Van Hove theorem, which requires that the Fermi energy is equal to the average single particle energy at saturation, we have the following relation [12]

$$\left(\frac{g_\omega}{m_\omega}\right)^2 = \frac{m - E_F^*(\rho_0) - \bar{B}}{\rho_0}, \quad (3)$$

where ρ_0 and \bar{B} are the saturation density and binding energy, respectively, and $E_F^* = (m^{*2} + k_F^2)^{1/2}$. Since the two EOS's in Ref. [9] have the same nucleon effective mass, the vector coupling constants are thus the same. At $3\rho_0$, the energy per nucleon with the “stiff” EOS is only about 15 MeV larger than that of the soft EOS. This is much smaller than the difference between the stiff and the soft EOS in the Skyrme parameterization (cf. dashed curves in Fig.1). As a result, the kaon yield is not very sensitive to the relativistic nuclear EOS's used in Ref. [9].

Up to $4\rho_0$ (see Fig. 1), the soft EOS used in Ref. [9] is very close to the soft EOS given by the Skyrme parameterization (Eq. (1)). To compare results between relativistic and non-relativistic approaches, we need to use a stiff EOS in the relativistic approach which is also similar to the stiff EOS used in the non-relativistic approach. Such an EOS can be obtained by using a smaller nucleon effective mass $m^* = 0.68 m$ but the same nuclear compressibility ($K=380$ MeV) at saturation density. The parameters for this EOS are

$$C_\sigma = 15.94, C_\omega = 12.92, B = 8.0 \times 10^{-4}, C = 2.26 \times 10^{-3}.$$

The two relativistic EOS's are shown in Fig. 1 by the solid curves. At $3\rho_0$, the two differ by about 56 MeV as in the Skyrme parameterization.

To see the sensitivity of subthreshold kaon production to the two relativistic nuclear EOS's, we have carried out a perturbative calculation of kaon production in Au+Au collisions using the relativistic transport model developed in Ref. [13]. Kaons are mainly produced from baryon-baryon interactions, and the production cross sections are taken from the linear parameterization of Randrup and Ko [14]. Contributions from meson-baryon interactions [15], higher resonances [16], and multi-baryon interactions [17] have been neglected as they

are unimportant for kaon production at energies around 1 GeV/nucleon. The rescattering of produced kaons with nucleons is treated by the perturbative test particle method introduced in [18]. Details of the calculations can be found in Ref. [9].

For a head-on Au+Au collision at 1 GeV/nucleon, we show in the left panel of Fig. 2 the total number of baryon-baryon collisions that have energies above the kaon production threshold. With the soft EOS, this number is about 95 but is reduced to about 54 when the stiff EOS is used. The reduction is partly due to the fact that the maximum central density reached with the soft EOS (about $2.9\rho_0$) is higher than that with the stiff EOS (about $2.4\rho_0$). As a result, the average density at which kaons are produced is also higher for the soft EOS (about $2.5\rho_0$) than for the stiff one (about $2.1\rho_0$). Furthermore, the energy per nucleon at these densities is about 5 MeV for the former and 15 MeV for the latter. Thus, more kinetic energy is converted into the compressional energy in the case of the stiff EOS. This effect can be seen from the the right panel in Fig. 2, where we show the distribution of p_{max} in the collision, with p_{max} being the maximum momentum of the produced kaon in a given baryon-baryon collision. The average value of p_{max} is about 0.272 GeV/c in the case of the soft EOS and is reduced to about 0.245 GeV/c for the stiff EOS. Overall, the kaon yield with the soft EOS is about a factor of two larger than that with the stiff one, consistent with the findings of non-relativistic transport models [6–8].

Our results thus demonstrate that the kaon yield from heavy ion collisions is similar in both relativistic and non-relativistic transport models if similar nuclear equations of state are used. In Ref. [2], the non-relativistic transport model was generalized to include a momentum-dependent potential, and it was shown that this would reduce significantly the kaon yield due to the lower number of collisions and deceleration as a result of the momentum-dependent potential. This result is in contrary to ours based on the relativistic transport model, which includes the momentum-dependent potential via the nucleon effective mass. We believe that the kaon yield calculated in Ref. [2] is incorrect as it has not taken into account the difference in the initial and final potential energies in the reaction $BB \rightarrow NYK$, where B and Y denote a baryon (nucleon or delta) and a hyperon (lambda

or sigma), respectively. Since baryons have larger momenta in the initial state than in the final state, some of the initial potential energy is available for kaon production and should compensate for the reduction of kaon yield due to the momentum-dependent potential. In our relativistic transport model [9], these effects are properly treated by including not only the nucleon effective mass but also the hyperon and kaon effective masses. The reduction of energy in the initial state of the reaction $BB \rightarrow NYK$ due to reduced nucleon in-medium mass is thus compensated by a corresponding reduction in the threshold as the final state energy is also reduced when in-medium masses are used. The net effect of modified hadron in-medium masses in the reaction is thus small, and our results are therefore similar to that from the normal non-relativistic transport model. However, this does not mean that it is correct to use the normal non-relativistic transport model to describe kaon production as we know that the nucleon mean-field potential is momentum-dependent and should be included. Furthermore, it is incorrect to assume, as in the non-relativistic transport model, that the hyperon has the same potential as the nucleon. From the phenomenology of hypernuclei, it is known that the mean-field potential for the hyperon is only about 2/3 of the nucleon potential [19,20]. Also, the neglect of kaon potential in the non-relativistic transport model is not warranted [21]. It has been recently shown that the kaon potential has significant effects on the flow of kaons in heavy ion collisions [22].

In Fig. 3 we compare the kaon momentum spectra obtained with the soft EOS (the same as reported in Ref. [9]) and the stiff EOS with the experimental data from the Kaos collaboration [4,5]. The theoretical results obtained with the soft EOS are in reasonable agreements with the experimental data, while that with the stiff EOS are below the data by about a factor of two. Thus, the Kaos data from Au+Au collisions at 1 GeV/nucleon favor a soft EOS. Because of the lack of empirical information on the elementary kaon production cross section from the nucleon-nucleon interaction near the threshold, this conclusion should be taken with some cautions [7].

As pointed out in Ref. [1], the difference between the kaon yields obtained with the soft and stiff EOS's in the Skyrme parameterization increases with decreasing incident energy.

To see whether this is also the case in a relativistic model, we have carried out calculations for head-on Au+Au collisions from 0.6 GeV/nucleon to 1.2 GeV/nucleon. The results are shown in Fig. 4. The left panel gives the kaon production probability P_{K^+} as a function of the incident energy $E_{\text{inc.}}$, and the right panel gives the ratio between the production probabilities obtained with the soft and stiff EOS's. We find that as the incident energy decreases from 1.2 GeV/nucleon to 0.6 GeV/nucleon, the ratio increases from 1.9 to 4.2, similar to the results from non-relativistic transport models [1]. The effect of the nuclear EOS on the kaon yield can thus be more clearly studied at lower incident energies.

In summary, using two relativistic EOS's that are similar to the Skyrme-type EOS's used in non-relativistic transport models, we have shown that in heavy ion collisions at subthreshold energies kaon production is sensitive to the nuclear EOS at high densities. Recent kaon data at 1 GeV/nucleon from the Kaos collaboration seem to favor a soft EOS. To learn more definitively about the nuclear equation of state at high densities, heavy ion experiments at lower incident energies, such as around 0.6 GeV/nucleon, will be very useful as kaon production at these energies is more sensitive to the nuclear EOS.

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Figure Captions

Fig. 1: Nuclear equation of state in the non-linear σ - ω model (solid curves) and in the Skyrme parameterization (dashed curves).

Fig. 2: Time evolution of the total number of baryon-baryon collisions that have energies above the kaon production threshold (left panel), and the distribution of the kaon maximum momentum p_{max} (right panel).

Fig. 3: Kaon momentum spectra obtained with the soft EOS (solid curve) and the stiff EOS (dashed curve). Experimental data from Ref. [5] are shown with open squares.

Fig. 4: Kaon production probability as a function of incident energy (left panel), and the ratio between the kaon production probabilities obtained with the soft and stiff EOS's.

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